

Experimental observation of chaotic hysteresis in Chua's circuit driven by slow voltage forcing

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ABSTRACT

The Chua's circuit has been considered as a paradigm for the investigation of chaos, but it still presents many unexplored dynamics. Chaotic hysteresis is an effect observed when the hysteresis phenomenon and chaotic dynamics phenomenon act simultaneously. It is an interesting phenomenon since it is observed in a variety of disciplines, and it has been utilized in explaining the mechanism of bursting oscillations, rich dynamics in thermal convection, and economic financial crisis, to mention a few. In this paper, we report the experimental observation of chaotic hysteresis in a Chua's electronic circuit driven by both a dc and a slow triangular voltage source. One and two hysteresis loops were observed in single and double scroll chaotic regimes, respectively. We present variations of the attractor that are induced by voltage changes, as well as the dependence of the hysteresis effect on both the driving frequency and the distance from the threshold passing from single to double-scroll chaotic regimes. Our results help in expanding our knowledge on unexplored dynamics of Chua's circuit as well as contribute to chaos control theory with a potential for applications.

1. Introduction

In the second half of the 20th century, the notion of chaos was introduced to characterize irregular, seemingly random dynamical behaviors of deterministic systems, that possessed the feature of sensitivity on initial conditions [5]. Since then, a lot of evidence of chaotic behavior by natural and artificial systems have been observed [6], while methods for detecting chaos in nature have been developed [7]. Chua's circuit was introduced in 1983 as the simplest chaotic system implemented by an electronic circuit, and it is the most experimentally studied chaotic electronic/dynamical system [8–10]. It is noteworthy that Chua's circuit has been used to build chaotic sensors [11,12], to show the possibility of control and synchronization of chaotic systems [13], and to present the stochastic resonance phenomenon [14].

In general, the hysteretic behavior of a system can be caused by lag between an input and an output, which disappears either due to slow input change or because of the system's state dependence on the direction of the input change [1]. The typical characteristic of ant hysteresis phenomenon is a looping behavior, *i.e.* the input-output characteristics

have the form of a loop. As a result, the dependence of the current system state on its history is observed and this is something emerging, among others, in mechanical and magnetic materials, even in biological and economic processes [2,3] and in different artificial systems [4].

It is well-known that chaotic dynamical behavior could emerge due to hysteresis effects [15,16]. Furthermore, the term *chaotic hysteresis* was introduced to describe a phenomenon when chaotic dynamics and hysteresis occur simultaneously. *Chaotic hysteresis* has been observed in thermal convection [17], neurons as bursting oscillations [18] and in the frequency-switched Chua's circuit [19], where several complex dynamics and hysteresis effects have been observed by changing the driving signal frequency. In recent years, chaotic dynamical behavior in Chua's circuit with dc-induced asymmetry has also attracted considerable attention. This asymmetry allowed for finding many new complex features such as mixed mode oscillations, bursting oscillations, as well as hidden attractors [20–22]. In [23], attractor folding and stretching, as well as, rich dynamics in Chua's circuit, when this is driven by a dc-voltage was numerically studied and presented.

The aim of this paper is to present the dependence of Chua's circuit

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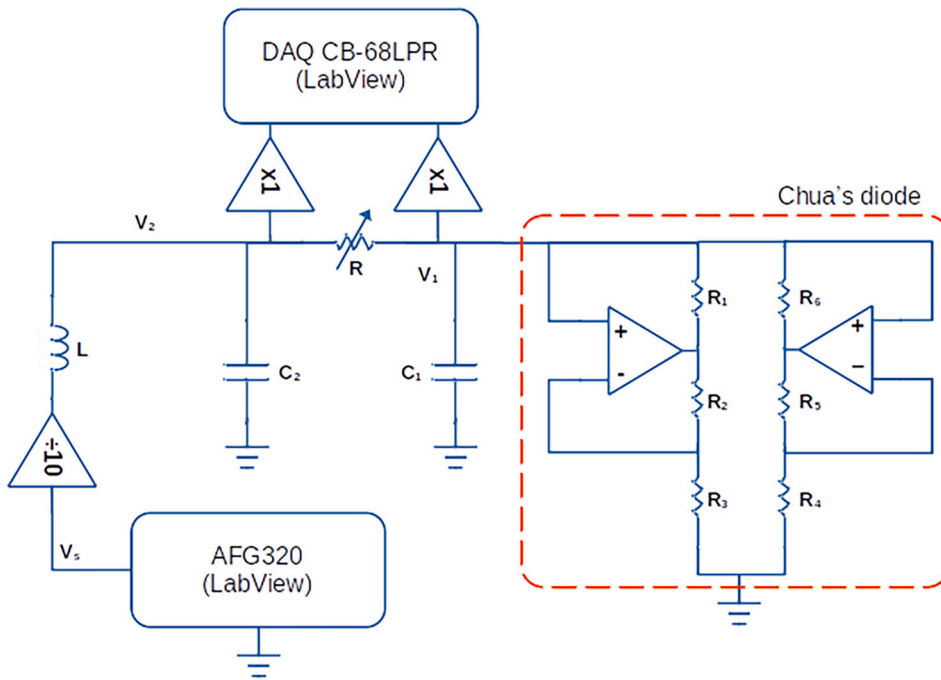


Fig. 1. Diagram of an experimental setup, including the implemented Chua's electronic circuit with the following electronic components: capacitors $C_1 = 10.2 \text{ nF}$ and $C_2 = 101.8 \text{ nF}$, a radial lead RF choke inductor $L = 17.52 \text{ mH}$ (RL181S series), operational amplifier *TL084CN*, the variable resistor R and Chua's diode resistors $R_1 = R_2 = 219 \Omega$, $R_3 = 2.185 \text{ k}\Omega$, $R_4 = 3.28 \text{ k}\Omega$, $R_5 = R_6 = 21.92 \text{ k}\Omega$. Voltage V_s is the output of the *AFG320* signal generator, and the DAQ board *CB - 68LPR* digitizes experimental data into the computer.

behavior on the direction of change of a quasi-dc variation. We show that a chaotic hysteresis in Chua's circuit is observed experimentally in the single scroll and double scroll chaotic dynamic regimes, as it is driven by both dc and a slowly varying triangular voltage. We present some hysteresis loops and describe changes of the attractor in the phase space. We define conditions for the occurrence of chaotic hysteresis, such as the forcing frequency range and the distance to the switching threshold between single- scroll and double-scroll chaotic regimes. Chua's circuit has proved to be very useful to study fundamental and applied issues of chaos theory. Our observations of the chaotic hysteresis effect in this circuit will contribute to a better understanding of this effect in other systems.

The paper is organized as follows: in Section 2, we present the experimental setup and describe all the details concerning our experiments. In Section 3, we describe the chaotic hysteresis produced by dc voltage in the single scroll chaotic regime. In Section 4, we present the chaotic hysteresis that is produced in this regime by a slow triangular voltage, and the change in the hysteresis loop caused by varying the driving frequency and the distance to the double scroll regime. In Section 5, we present the chaotic hysteresis resulting from both dc and a slow triangular voltage driving, demonstrating two loops and observed in the double scroll chaotic regime, close to the threshold between single and double scroll. Finally, we provide conclusions in Section 6.

2. Experimental setup and procedure

The experimental setup consists of an implementation of Chua's electronic circuit [8–10], a DAQ *CB-68LPR* controlled by Labview to acquire experimental data, and a Sony function generator *AFG320* used to provide the driving voltage. This setup is presented in Fig. 1. The Chua's electronic circuit was assembled according to [24], using the suggested electronic components, as indicated in Fig. 1, using an external power supply at $\pm 15 \text{ V}$ for the Operational Amplifiers. The variable resistor R was used to tune the different dynamics of the system. By varying the resistance of R , Chua's circuit exhibits a sequence of bifurcations that range from an equilibrium state, to periodic, period-doubling, single scroll or double scroll regimes [9,10]. These regimes were observed in the system's phase portraits (V_1 , V_2) on the oscilloscope, where V_1 and V_2 are shown in Fig. 1. It should be mentioned that we have ensured that the circuit oscillations were always far from the Operational Amplifiers saturation range.

In our experiments, we first picked the value of R to set Chua's circuit operating region in the single or double scroll chaotic regime, in the absence of external driving. The chaotic attractors in these regimes are shown in Fig. 2. In the single scroll regime, the system can reside within two distinct basins of attraction depending on the initial conditions. Each single scroll attractor is characterized by irregular oscillations and the trajectory winds around a fixed point with a dominant frequency determined by the values of the components, which in our case was 2.985 kHz . In Fig. 2 the oscillation direction denoted by $P+$ is clockwise

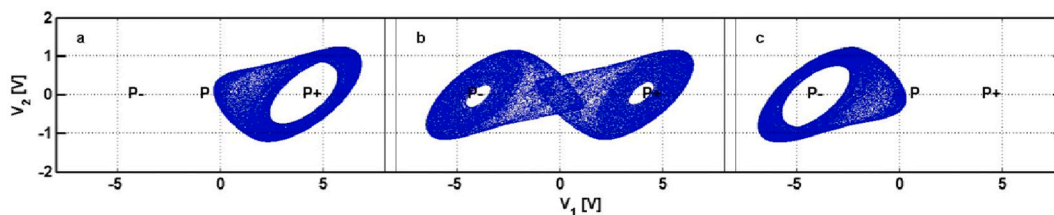


Fig. 2. The phase portraits of the demonstrated attractors of the Chua's circuit as these are observed experimentally, in the absence of external voltage, with clockwise and counterclockwise oscillations around fixed points $P+$ and $P-$. Insets (a) and (c) correspond, respectively, to clockwise and counterclockwise oscillations in the single scroll regime for $R = 1725 \Omega$, while (b) shows the double scroll regime for $R = 1702 \Omega$.

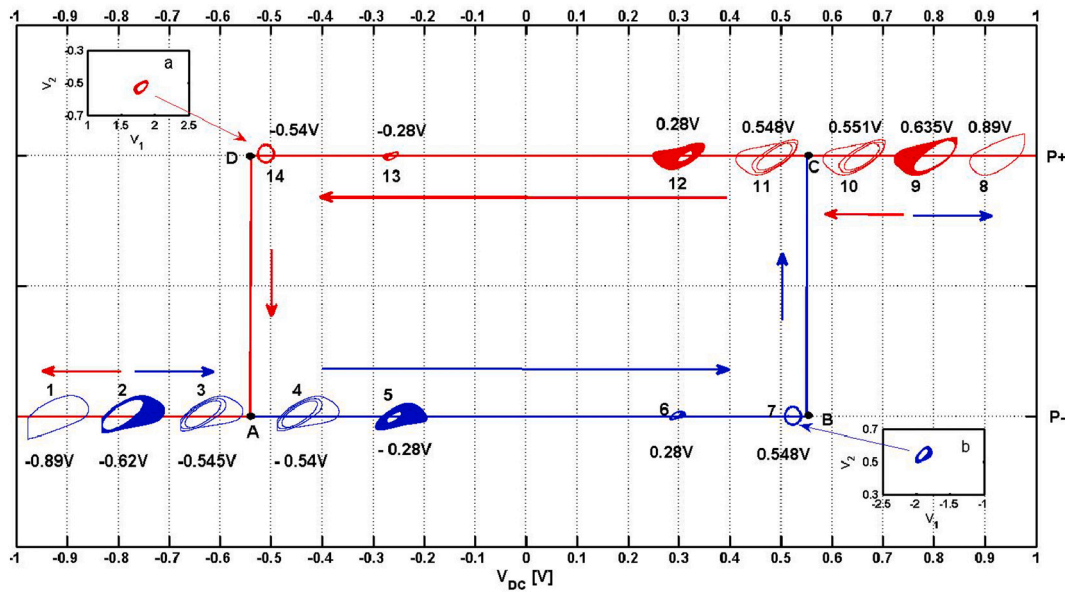


Fig. 3. The chaotic hysteresis phenomenon in Chua's circuit, caused by a very slow step variation of the constant voltage V_{DC} . $P+$ and $P-$ are fixed points for clockwise and counterclockwise oscillations shown in Fig. 2. The blue and red paths and arrows correspond to increasing and decreasing V_{DC} , respectively. Points A, B, C, D define the hysteresis loop. Numbers 1–14 denote selected values of V_{DC} provided in the figure with the corresponding form of attractors plotted in the same region of the phase space. The attractors in points 7 and 14 are plotted separately because of their small size (insets b and a, respectively). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

around the fixed point, whereas the one denoted by $P-$ is counterclockwise. In the double scroll regime the attractor consists of two coexisting scrolls, located around the two fixed points $P+$ and $P-$ connected by fast jumps between them. After setting the Chua's circuit dynamics in the absence of any external driving, we further controlled its dynamic behavior by applying an external driving voltage V_s . We used two different configurations: one using a direct current voltage $V_s = V_{DC}$, and another using a triangular signal $V_s = V_T$.

3. Experimental observation of chaotic hysteresis produced by DC voltage

Initially, we set R to $1.725\text{ k}\Omega$; in the absence of external voltage the circuit operates in the chaotic regime with the single scroll attractor and the dynamic trajectory oscillating clockwise around fixed point $P+$, as shown in Fig. 2a. Then, we added a constant voltage $V_s = V_{DC}$ and changed its value very slowly and step by step through minimal changes in the voltage generated by the AFG320 voltage source. We started by

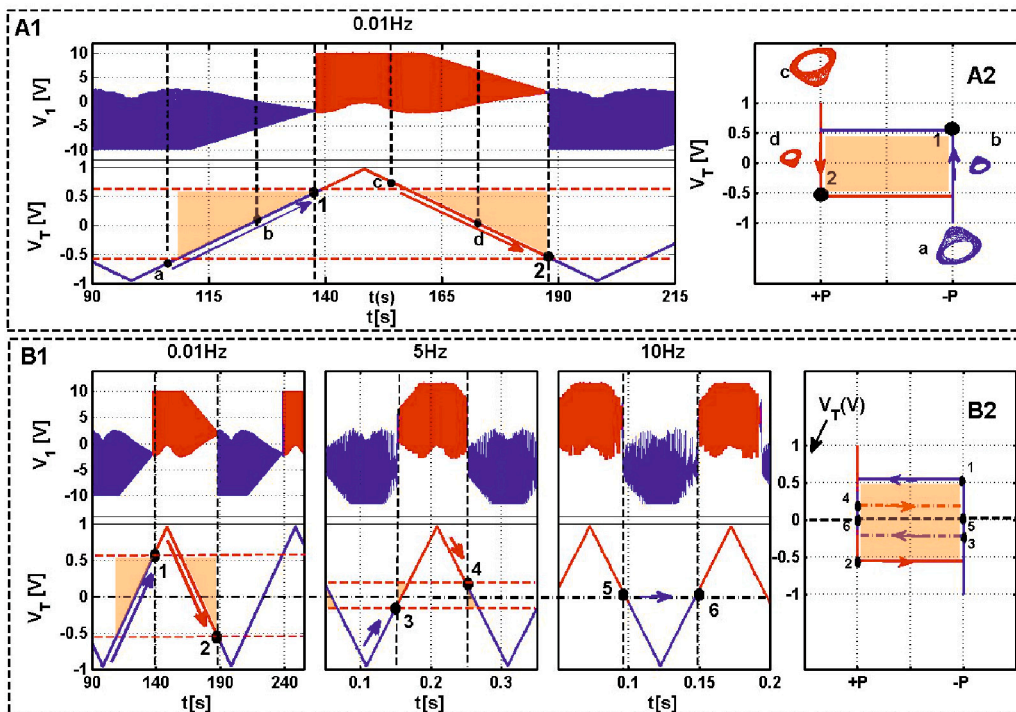


Fig. 4. Time series of the voltage V_1 on capacitor $C1$ and triangular voltage forcing V_T with the amplitude 1 V and the frequency 0.01 Hz (A1) and frequencies 0.01 Hz , 5 Hz and 10 Hz (B1) observed in Chua's circuit for $R = 1.725\ \Omega$. The red and blue colors mark data corresponding respectively to oscillations of dynamic trajectory around fixed points $P+$ and $P-$ shown in Fig. 2. The insets A2 and B2 show relations between the value of V_T and selected fixed point for dynamic trajectory oscillations, which correspond respectively to the time series given in A1 and B1. In these insets the blue and red paths and arrows correspond to increasing and decreasing V_T respectively. The inset A2 shows phase portraits of attractors at points a–d of time series A1. In all insets the numbered points, dashed red horizontal lines and orange regions mark the range of V_T and time intervals in which the system is in a state depending on its history. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

decreasing V_{DC} from 0 V to -1 V and we observed the system's dynamic behavior in the phase portrait (V_1, V_2) by increasing and decreasing $V_s = V_{DC}$ in the range $[-1\text{ V}, 1\text{ V}]$. Our observations are presented in Fig. 3. In this figure we show the system attractors for selected values of $V_s = V_{DC}$, assuming ranges $[-10\text{ V}, 10\text{ V}]$ and $[-4\text{ V}, 4\text{ V}]$ for V_1 and V_2 respectively. It is apparent that there exist periodic and chaotic regimes with single scroll attractor, while bifurcations between these states were also observed. The absence of any double scroll attractor is worth to be noted.

In the same Fig. 3, we also identified the effect of chaotic hysteresis by lines, colors and arrows, with the loop marked with points denoted by letters A, B, C and D. The path for increasing voltage V_{DC} is marked in blue, while the path for decreasing V_{DC} is marked in red. The path 1–7 starts with the dynamic trajectory oscillating around point P–, then the attractor shrinks up to the point B, corresponding to $V_{DC} = 0.548\text{ V}$, where it abruptly expands and the dynamic trajectory begins to oscillate around point P+. This effect is well visualized by attractors in points 7 and 10. Further increasing V_{DC} only changes the attractor state between periodic and chaotic mode of operation, around the fixed point P+ and finally the system ends in a steady state.

The path for decreasing voltage V_{DC} (in red) is similar. The attractor undergoes bifurcations between periodic and chaotic states and shrinks. At the point D, which corresponds to $V_{DC} = -0.548\text{ V}$ the system abruptly expands, changing its state from chaotic to periodic (attractors at points 14 and 3) and the direction of oscillations from clockwise around fixed point P+ to counterclockwise around fixed point P–. It should be noted that the observation of the described hysteresis effect is possible only with very slow step variation of a constant voltage V_{DC} . Notice that the attractor's stretching and folding caused by variations of the constant voltage was also observed and described in [23].

There is a range of constant driving voltage V_{DC} (between points A and B in Fig. 3) where two very different dynamic states of the system are possible. The attractors in points 5 and 13 for $V_{DC} = -0.28\text{ V}$ and in points 7 and 11 for $V_{DC} = +0.548\text{ V}$ are very different. For the same V_{DC} the system's trajectory can oscillate around different fixed points in opposite directions. The state of the system for a given value of DC voltage can be chaotic with a single scroll attractor or periodic, depending on how this voltage changed in the past. This dependence of the state of Chua's circuit on its history is a sign of the hysteresis effect. For voltages V_{DC} outside the hysteresis loop the dynamics of the system does not depend on the direction of its changes.

4. Experimental observation of chaotic hysteresis produced by slow triangular forcing

The hysteresis described in the previous section has been observed for a very slow step variation of a DC driving voltage. In order to determine how slow these variations may be, we applied a triangular voltage forcing $V_s = V_T$ at very low frequencies (0.01, 5, 10 Hz). This way, we could determine the frequency dependence of the observed hysteresis. In this second experiment, without any external signal, Chua's circuit was in the single scroll regime and the resistance R was set to $1.725\text{ k}\Omega$ as in the previous section. The data acquisition sampling frequency was 10 kHz for the triangular signal at the frequency 0.01 Hz , while a sampling at 20 kHz was needed for the triangular signals at 5 Hz and 10 Hz . The system's resulting behavior is depicted in Fig. 4.

In this figure, the two different states of the system are well represented by the dependence V_T . The red color on this time series data denotes one state of the system in which its dynamic trajectory oscillates around the fixed point P+, while the blue color denotes another system state in which the dynamic trajectory oscillates around the fixed point P–. The abrupt changes between these two states and different attractors are marked by numbered points and dashed red horizontal lines on the corresponding time series V_T . The orange regions denote the range of the triangular voltage and time intervals in which the system can be in a state depending on how V_T changed in the past. For a very low frequency

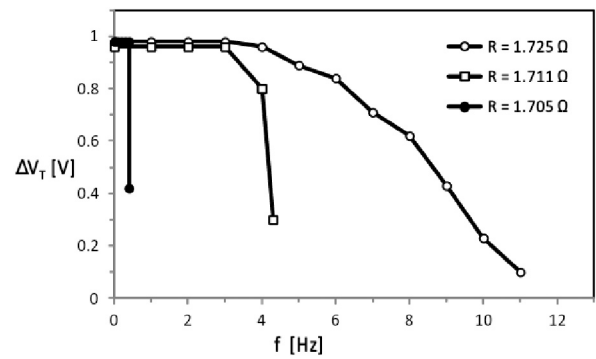


Fig. 5. The dependence of triangular voltage range ΔV_T in which Chua's circuit can be in two states depending on its history on the forcing frequency for different values of this circuit resistivity R given in the figure.

0.01 Hz of V_T , when V_T increases from -1 V to 1 V (marked by blue arrow) the attractor concentrated around the fixed point P– shrinks until a limit around $V_T = +0.5\text{ V}$ (point 1 in Fig. 4). At this limit, the system dynamic trajectory abruptly jumps from a region of phase space near P– where its attractor is very small and chaotic to the region near P+ where it forms a large periodic attractor.

An analogous process occurs when the voltage V_T decreases from $+1\text{ V}$ to -1 V . There is an abrupt jump of the system dynamic trajectory at $V_T \approx -0.5\text{ V}$ (point 2 in Fig. 4) from a region of phase space near P+, where it forms a very small chaotic attractor, to the region near P– where the attractor is large and periodic. In the vicinity of points 1 and 2, two dynamics of the system, chaotic and periodic, are adjacent. The same is in the vicinity of lines A–D and B–C in Fig. 3. For the frequency 0.01 Hz of V_T and its value between the two limits the dynamic trajectory can oscillate clockwise around fixed point P+ or counterclockwise around fixed point P– depending on how V_T changed in the past. Outside these limits, the dynamics of the system is independent of the direction of V_T changes. Our experiment thus showed evidently the chaotic hysteresis effect in Chua's circuit for very slow voltage driving. It should be noted that this external excitation did not induce the formation of a double scroll attractor.

In order to study the effect of faster variations of voltage forcing on the hysteresis effect we applied a triangular voltage waveform at higher frequencies. The results are shown in Figs. 4 and 5. Let's denote by ΔV_T the voltage range in which Chua's circuit can be in one of two states depending on how V_T changed in the past. One can notice that for frequencies smaller than 4 Hz the range ΔV_T is independent of the driving frequency. For higher frequencies it smoothly decreases with the inflection point around 8 Hz and for $f > 10\text{ Hz}$ the hysteresis phenomenon practically disappears and the double scroll attractor becomes visible on the oscilloscope. The effect of reducing the hysteresis range ΔV_T with the increase in the forcing frequency was observed and discussed in [1,19].

For values of the resistance R higher than $1.725\text{ k}\Omega$, we observed in the single scroll regime a very similar hysteresis effect. For $R < 1.725\text{ k}\Omega$, our circuit without external forcing was still in the single scroll regime, and for $R \leq 1.704\text{ k}\Omega$ it was in the double scroll regime. In Fig. 5 we show the dependence of the voltage range ΔV_T on the forcing frequency for the three values of R : $1.725\text{ k}\Omega$, $1.711\text{ k}\Omega$ and $1.705\text{ k}\Omega$. One can notice that close to the double scroll regime the triangular voltage range ΔV_T drastically decreases and very near this regime the chaotic hysteresis phenomenon disappears. Moreover, there is a sharp drop in the dependence of ΔV_T on the triangular voltage forcing frequency close to the double scroll regime.

5. Double chaotic hysteresis produced by slow triangular forcing

In our experiments for $R \leq 1.704\text{ k}\Omega$, Chua's circuit without any

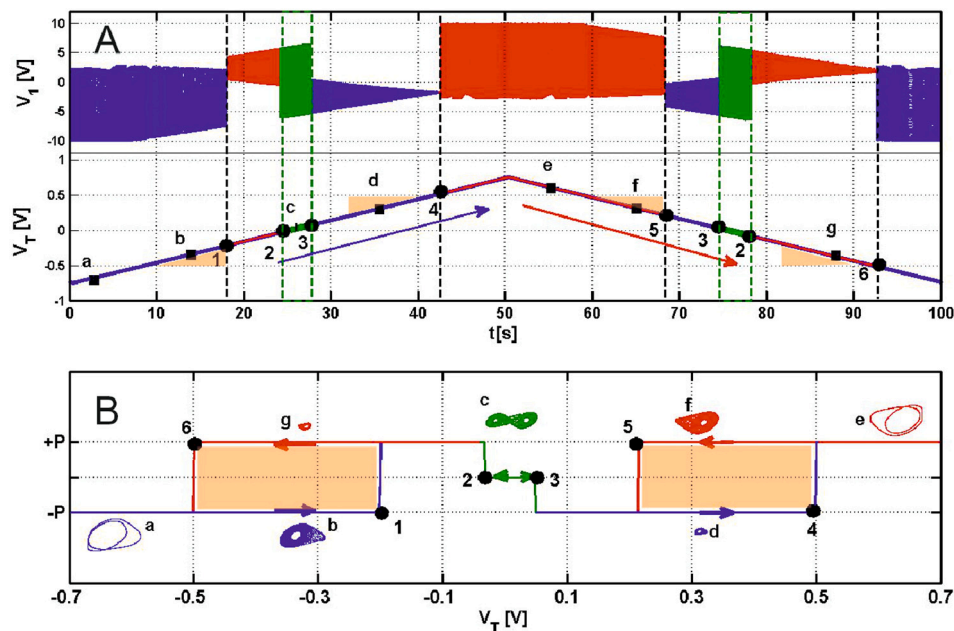


Fig. 6. Time series of the voltage V_1 on capacitor $C1$ and external triangular voltage forcing V_T with the amplitude 1 V and the frequency 0.01 Hz observed in Chua's circuit with $R = 1.702\ \Omega$ (inset A). The red and blue colors mark data corresponding respectively to oscillations of dynamic trajectory around fixed points P_+ and P_- —shown in Fig. 2, whereas green color mark data corresponding to oscillations of dynamic trajectory on double scroll attractor. The insets B shows relations between the amplitude of triangular forcing and the selected fixed point for dynamic trajectory oscillations with phase portraits of attractors at points a–g of time series V_T . In this inset the blue and red paths and arrows correspond to increasing and decreasing forcing amplitude respectively. The orange regions mark the range of V_T and time intervals in which the system is in a state depending on its history. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

external excitation was in the double scroll dynamical regime. In this regime for values of R too far from the threshold $R = 1.704\text{ k}\Omega$ the chaotic hysteresis effect was not experimentally observed. However, when the resistance R was very close to the threshold for $R = 1.702\text{ k}\Omega$, and a triangular voltage driving was used, we observed chaotic hysteresis with two loops *i.e.* the double hysteresis. In Fig. 6 we present experimental results obtained for a triangular voltage V_T at very low frequency 0.01 Hz and amplitude 1 V . The double scroll attractor, marked in green in this figure, is observed only for very small values of the driving amplitude. For higher values of this amplitude, the double scroll attractor is destroyed and there are oscillations of the dynamic trajectory around the fixed points P_+ for $V_T < 0$ or P_- for $V_T > 0$. Further increase of V_T leads to two hysteresis loops, one for positive and the other for negative values of the driving voltage. There are thus two ranges of the external driving, with different dynamical behavior of Chua's circuit, depending on how this external excitation changed in the past. Within these ranges, the dynamic trajectory oscillates clockwise or counterclockwise around different fixed points P_+ or P_- . This oscillation varies between chaotic and periodic very similar as it was described in Section 2 for the hysteresis observed in the single scroll regime. For very high amplitudes of forcing, the system is in one state with dynamic trajectory oscillating on a single scroll attractor which changes between chaotic or periodic depending on the forcing strength. We observed the same dynamical behavior and the double hysteresis when using a dc driving voltage.

6. Conclusions

In the sections above, we have presented the experimental observation of the chaotic hysteresis effect in a Chua's circuit. This hysteresis has been observed in both the single or the double-scroll regimes of operation, always when it is driven by dc and slow triangular voltage source inserted into its inductor branch. In the single scroll region, there is a single hysteresis loop. In this case the voltage range of this loop quickly decreases with decreasing the distance to the double scroll regime. Further on, passing through the threshold between the single and double scroll regime, the dependence on the forcing frequency of the loop voltage range is constant up to around 4 Hz ; then it gradually decreases to zero at about 10 Hz . Near the double scroll regime, this dependence is constant in a small voltage range and then it drops sharply. In the double scroll chaotic regime, the chaotic hysteresis exists

only very near the single scroll regime threshold. The hysteresis consists of two loops for the positive and negative voltage driving *i.e.* there is a double hysteresis. All hysteresis loops observed in our experiments consist of two segments ending with a quick transition between chaotic and periodic mode of operation. Along these segments, the dynamic behavior of the system undergoes bifurcations and thus, the dynamic attractor folds and stretches. We believe that our results contribute to further explore the dynamics of a well-studied dynamical system and circuit, further contributing to the development of chaos theory. Additionally, these results may find application in chaos control, either for suppression of unwanted behavior or for its utilization in interdisciplinary applications as, for instance, communications.

CRedit authorship contribution statement

I.G., W.K. conceived the paper's idea; I.G., W.K., and R.P. devised the methodology and coordinated the work; I.G., W.K., S.G.S. and R.P. performed the experimental part, I.G., W.K., S.G.S., R.P. and L.O.C. analyzed the results; All authors participated in writing and reviewing the manuscript.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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